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## Gauge Coupling Unification due to Large String Threshold Corrections

EDI HALYO<sup>\*</sup>

Department of Particle Physics  
Weizmann Institute of Science  
Rehovot 76100, Israel

### ABSTRACT

We show that large string threshold corrections can reconcile the string and MSSM unification scales in fermionic strings. This requires at least three moduli with large VEVs which are different from each other and MSSM states arising in an unconventional manner from the string spectrum. The former is easily achieved by supersymmetry breaking by both hidden gaugino and matter condensation whereas the latter needs to be seen in explicit string models.

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\* e-mail address: jphalyo@weizmann.bitnet, Address after September 1: Department of Physics, Stanford University, Stanford CA 94305.

## 1. Introduction

The unification of the  $SU(3) \times SU(2) \times U(1)$  gauge couplings is considered to be a great achievement of the minimally supersymmetric Standard Model (MSSM)[1]. Assuming only the MSSM spectrum without any additional states, one finds that the three gauge couplings unify around  $M_U \sim 2.5 \times 10^{16} \text{ GeV}$  with  $g_U \sim 0.7$ . This result strengthens the belief in both unification and supersymmetry.

The most fundamental theory of high energy physics to date is superstring theory which gives a unified description of all interactions including gravity[2]. It is thus natural to try to realize the ideas of unification of gauge couplings and supersymmetry in superstring theories. Whereas supersymmetry naturally arises from the superstring, the situation is different for the unification of gauge couplings. In the string context, one assumes that the three gauge couplings unify around the string (or Planck scale) due to the fact that at that scale all interactions are different aspects of the only string interaction. In fact, it has been shown that gauge couplings of the string unify around  $M_S \sim g_S \times 5.5 \times 10^{17} \text{ GeV}$  where  $g_S \sim 0.7$  is the string coupling at that scale[3]. Thus one finds that there is an order of magnitude discrepancy between the MSSM and string unification scales which needs to be explained. Conversely, if one takes the string unification scale and runs down the gauge couplings with only the MSSM spectrum, one obtains results for  $\sin^2\theta(M_Z)$  and  $\alpha_3(M_Z)$  which are in conflict with experiments.

There have been different attempts to explain the above discrepancy. Among them one can count introducing additional states at intermediate energies[4], separating the soft supersymmetry breaking masses around the TeV scale[5] and including the string threshold corrections[3,6]. The former two depend on intermediate and low-energy physics (compared to the Planck scale) and are therefore strongly model dependent. The string threshold corrections, on the other hand, offer an elegant stringy solution to the problem around the Planck scale without invoking new particles and/or physics. In addition, since these corrections are fixed for a given string model they are much more constrained. Unfortunately, in free fermionic

(and orbifold) string models built up to date, the threshold corrections are not large enough to solve the problem due to the small (i.e.  $\sim 1$ ) overall modulus VEV which is a result of supersymmetry breaking by hidden gaugino condensation [7,13]. In addition, in free fermionic models, the sign of the threshold corrections is wrong; i.e. they increase the discrepancy rather than decrease it[4].

In this letter, we consider unification of the gauge couplings due to large string threshold corrections in fermionic strings[8]. We show that in order for these to reduce the string unification scale  $M_S$  down to  $M_U$  or to give realistic values for  $\sin^2\theta_w(M_Z)$  and  $\alpha_3(M_Z)$ , there must be at least three moduli with large ( $>> 1$ ) VEVs which are different from each other. Moduli VEVs different than unity correspond to a fermionic string deformed marginally by Abelian Thirring interactions whose couplings are related to the VEVs themselves. In addition, the MSSM states must arise from the different sectors of the string spectrum in an unconventional manner so that the threshold corrections have the correct sign. In section 2, we briefly review string threshold corrections and why they do not offer a solution in realistic free fermionic string models built so far. In section 3, we review supersymmetry breaking in the presence of hidden gaugino and matter condensation and how this scenario results in large moduli VEVs as required. In section 4, we show how to get the desired threshold corrections which give the correct  $\sin^2\theta_w(M_Z)$  and  $\alpha_3(M_Z)$  (or reduce the string unification scale  $M_S$  down to  $M_U$ ). Section 5 contains our conclusions.

## 2. String Threshold Corrections

The one-loop renormalization group equations (RGE) for the gauge couplings including the string threshold corrections are given by[3]

$$\frac{16\pi^2}{g_a^2(\mu)} = k_a \frac{16\pi^2}{g_S^2} + b_a \log \frac{M_S^2}{\mu^2} + \Delta_a \quad (1)$$

where  $a = 1, 2, 3$  corresponding to  $U(1)_Y, SU(2)_L, SU(3)_C$  respectively.  $b_a$  and  $k_a$  are the MSSM  $\beta$ -function coefficients and the level of the corresponding Kac-Moody algebras. In all string models built to date,  $k_2 = k_3 = 1$  whereas  $k_1$  can be

considered as a free parameter [9].  $\Delta_a$  are the string threshold corrections to the running of gauge couplings which arise from the infinite tower of massive string states. They can be divided as  $\Delta_a = \tilde{\Delta}_a + c_a + Y$  where  $\tilde{\Delta}_a$  gives the contributions which depend on the untwisted moduli of the string model. The VEVs of these moduli are free in perturbation theory to all orders but are fixed by nonperturbative effects such as condensation in the hidden sector which also break supersymmetry.

Untwisted moduli in free fermionic models have been examined extensively in Ref. [10]. Depending on the boundary condition vectors defining the string model there can be up to three moduli of the  $T$  and  $U$  types, each pair corresponding to one of the three compactified tori. Every left-right asymmetric boundary condition or complex world-sheet fermion eliminates two moduli so that there can be six, four, two or no moduli in these models. In the section 4, we will see that large threshold corrections which are phenomenologically acceptable require the presence of at least one modulus from each torus. The string spectrum can be divided into three different parts which have  $N = 4, 2, 1$  supersymmetry respectively. Only the sectors with  $N = 2$  supersymmetry contribute to  $\tilde{\Delta}_a$ [3,6]. In realistic free fermionic models there are three sectors with  $N = 2$  supersymmetry which give one generation of fermions each[4]. These sectors also correspond to the twisted sectors of the string, each giving one of the three twists of the  $Z_2 \times Z_2$  which forms the basis of realistic free fermionic strings. The moduli dependent part of the threshold effects is given by

$$\tilde{\Delta}_a = - \sum_i \frac{1}{2} b_a'^i \log [Re T_i |\eta(T_i)|^4 Re U_i |\eta(U_i)|^4] \quad (2)$$

Here  $b_a'^i$  are the  $N = 2$   $\beta$ -function coefficients for the gauge group fixed by  $a$  and the  $N = 2$  sector  $i = 1, 2, 3$ .  $T_i$  in Eq. (2) are the moduli corresponding to the torus left fixed under the twist which defines the  $N = 2$  sector.  $\eta(T) = e^{-\pi T/12} \Pi_n (1 - e^{-2\pi nT})$  is the Dedekind  $\eta$ -function. This one-loop expression is exact due to the  $N = 2$  nonrenormalization theorems. The small universal piece  $Y$  turns out to be not important since one can absorb it into the definition of  $g_S$ .

The gauge dependent piece  $c_a$  does not depend on untwisted moduli and receives contributions from only the  $N = 2$  supersymmetric sector in free fermionic strings. Generically  $c_a$  are very small in free fermionic strings [11] so that they do not affect the results for quantities such as  $\sin^2\theta_w(M_Z)$  and  $\alpha_3(M_Z)$  significantly. We neglect them in the following. Using Eqs. (1) and (2) we find that

$$\sin^2\theta_w(M_Z) = \frac{k_1}{k_1 + k_2} + \frac{\alpha_1(M_Z)}{4\pi} \frac{k_2}{k_1 + k_2} [A \log(\frac{M_Z^2}{M_S^2}) + \Delta_A] \quad (3)$$

and

$$\alpha_3(M_Z)^{-1} = \frac{k_3}{k_1 + k_2} [\frac{1}{\alpha_1(M_Z)} + \frac{B}{4\pi} \log(\frac{M_Z^2}{M_S^2}) + \frac{\Delta_B}{4\pi}] \quad (4)$$

where

$$A = (b_1 \frac{k_2}{k_1} - b_2) \quad B = (b_1 + b_2 - b_3 \frac{(k_1 + k_2)}{k_3}) \quad (5)$$

and

$$\Delta_A = -(\Delta_1 \frac{k_2}{k_1} - \Delta_2) \quad \Delta_B = -(\Delta_1 + \Delta_2 - \Delta_3 \frac{(k_1 + k_2)}{k_3}) \quad (6)$$

Here  $\alpha_1(M_Z) = (127.9 \pm 0.1)$  is the electromagnetic structure constant at the weak scale and  $b_{1,2,3} = 11, 1, -3$  are the  $\beta$ -function coefficients for the MSSM spectrum.  $k_2 = k_3 = 1$  whereas  $k_1$  is a free parameter. From Eq. (1) for the running coupling constants we can also obtain the unification scale,  $M_T$ , in the presence of threshold corrections

$$M_T = M_S \prod_i [\sqrt{ReT_i} |\eta(T_i)|^2 \sqrt{ReU_i} |\eta(U_i)|^2]^{b_2^{ii'} - b_3^{ii'} / b_3 - b_2} \quad (7)$$

Neglecting the threshold corrections and using  $g_S \sim 0.7$  and  $M_S \sim g_S \times 5.5 \times 10^{17} \text{ GeV}$  we get  $\sin^2\theta_w(M_Z) = 0.2187$  and  $\alpha_3(M_Z) = 0.195$  which do not agree with the experimental values  $\sin^2\theta_w(M_Z) = 0.2319 \pm 0.0005$  and  $\alpha_3(M_Z) = 0.120 \pm 0.007$ [12]. The problem of string unification can be formulated in two equivalent ways: without the threshold corrections either  $M_T (= M_S)$  is an order

of magnitude larger than  $M_U$  or equivalently, with the values of  $M_S$  and  $g_S$  given above, one finds too small (large) a value for  $\sin^2\theta_w(M_Z)$  ( $\alpha_3(M_Z)$ ). The latter is simply the result of the extra running of the gauge couplings from  $M_S$  which is an order of magnitude larger than  $M_U$ .

Can the string threshold corrections make up for the difference? Considering Eq. (2) for  $\tilde{\Delta}_a$ , it is usually assumed that there is only an overall modulus  $T$  for simplicity even though most string models have more than one untwisted moduli as we mentioned above. The VEV of  $T$  in  $\tilde{\Delta}_a$  is fixed by hidden sector condensation effects which also break supersymmetry. In the scenario with supersymmetry breaking by hidden gaugino condensation one obtains  $T \sim 1$ [13]. Using the relation  $\sum_i \frac{1}{2}b_a'^i = b_a$  which holds for free fermionic strings[14], one finds that  $\tilde{\Delta}_a$  are too small to make up for the difference between  $M_S$  and  $M_U$ . In addition, in free fermionic string models,  $\tilde{\Delta}_a$  are such that the sign of the corrections  $\Delta_A$  and  $\Delta_B$  is wrong; i.e. they decrease  $\sin^2\theta_w(M_Z)$  and increase  $\alpha_3(M_Z)$  rather than the opposite.

Equivalently, one finds that threshold corrections give through Eq. (7),  $M_T > M_S$  i.e. they increase  $M_T$  rather than decrease it to  $M_U$ . This has also been established by explicit numerical calculations of  $\tilde{\Delta}_a$  in fermionic strings. The reason for this lies in the fact that in fermionic strings all matter have modular weights (which are related to the R charges of matter fields[15]) equal to  $-1$  under target space duality of the overall modulus  $T$ . This leads to the relation  $\sum_i \frac{1}{2}b_a'^i = b_a$  which together with  $b_a$  for the MSSM spectrum give the wrong sign. Thus, in order to explain the discrepancy between  $M_S$  and  $M_U$  or obtain experimentally acceptable values of  $\sin^2\theta_w(M_Z)$  and  $\alpha_3(M_Z)$  only by string threshold corrections we need a) large VEVs for moduli so that the magnitude of  $\tilde{\Delta}_a$  is large b) switch the sign of  $\tilde{\Delta}_{A,B}$ .

### 3. Supersymmetry breaking due to gaugino and matter condensation

We have seen that the moduli VEVs must be large ( $>> 1$ ) in order to obtain large string threshold corrections. In this section, we show that if supersymmetry is

broken by hidden matter condensation in addition to hidden gaugino condensation the vacuum is at large  $ReT_i$  as required. This is in contrast to the pure gaugino condensation case with only an overall modulus in which the vacuum is given by  $ReT \sim 1.22$  [13] which is not large enough.

When the hidden gauge group ( $SU(N)$ ) of a superstring (or supergravity) becomes strongly interacting gaugino condensates,  $Y^3$ , form. If there is also hidden matter ( $M_i$ ), as it is the case in generic string models, then matter condensates  $\Pi_{ij} = \langle M_i \bar{M}_j \rangle$  form in addition to  $Y^3$ . The effective superpotential which describes the low-energy effective theory after condensation is given by[16]

$$W_{eff} = \frac{1}{32\pi^2} Y^3 \log\{\exp(32\pi^2 S)[c\eta(T)]^{6N-2M} Y^{3N-3M} \det\Pi\} - \text{tr} A\Pi, \quad (8)$$

where  $c$  is a constant and  $A$  is the hidden matter mass matrix which must be nonsingular in order to have a stable vacuum[17].  $S$  and  $T$  are the dilaton and overall modulus respectively. For simplicity here we consider only one modulus. The generalization to more than one modulus is straightforward.

Taking the flat limit  $M_P \rightarrow \infty$  one eliminates the strongly interacting condensates  $Y^3$  and  $\Pi$  and obtains the effective superpotential in terms of  $S$  and  $T$

$$W_{np}(S, T) = \Omega(S) h(T) [\det A]^{1/N}, \quad (9)$$

where

$$\Omega(S) = -N \exp(-32\pi^2 S/N), \quad (10a)$$

$$h(T) = (32\pi^2 e)^{M/N-1} [c\eta(T)]^{2M/N-6}. \quad (10b)$$

$\det A$  is generically given by[18,19,20]

$$\det A = k(ReS)^{-r} \phi_j^{s_j} \eta(T)^t \quad r, s, t > 0, \quad (11)$$

where the  $S$  dependence is obtained from the relation  $g^2 = 1/4ReS$ ,  $k$  is a constant and  $\phi_j$  are Standard Model scalar singlets whose VEVs give mass to the

hidden matter. The power of  $\eta(T)$  is fixed by the requirement that individual mass terms be modular invariant. Using Eq. (9) for the superpotential and the Kahler potential

$$K = -\log(2ReS) - 3\log(2ReT) - \sum_j (2ReT)^{n_j} \phi_j \phi_j^\dagger \quad (12)$$

we obtain the effective scalar potential[19]

$$\begin{aligned} V_{eff} = & \frac{e^{-\phi_j \phi_j^\dagger / 2ReT}}{16ReS(ReT)^3 |\eta(T)|^{8\pi d'}} |[detA]^{1/N}|^2 \{ |2ReS\Omega_S - \Omega - \frac{2\Omega r}{N}|^2 \\ & + |\Omega|^2 \left( \frac{4d'^2(ReT)^3}{(3ReT - \phi_i \phi_i^\dagger)} |G_2(T) - \frac{3}{2ReTd'} + \frac{\phi_j \phi_j^\dagger}{4(ReT)^2 d'}|^2 - 3 \right) \}. \end{aligned} \quad (13)$$

where  $d' = (6N - 2M - t)/4\pi N$ . Here  $G_2$  is defined through the derivative of  $\eta(T)$  as  $\partial\eta(T)/\partial T = -\eta(T)G_2(T)/4\pi$ . Comparing  $V_{eff}$  above to that of the pure gaugino case we see that the effect of hidden matter condensates and their mass terms is simply to change the function  $\hat{G}_2(T) = G_2 - \pi/ReT$  in the pure gaugino case to  $G_2(T) - 3/2ReTd' + \phi_j \phi_j^\dagger / 4(ReT)^2 d'$  where  $d'$  is fixed by the hidden gauge group ( $N$ ), the matter content of the hidden sector ( $M$ ) and the hidden mass terms ( $t$ ) in Eq. (11).

The potential above was studied in detail in Ref. [19]. The results are as follows. As  $M$  and/or  $t$  increase (which corresponds to more and/or lighter hidden matter) so that  $d'$  decreases,  $T_R$  at the minimum increases from 1.22 which is the value obtained from pure gaugino condensation. For example  $d' = 1/2\pi$  and  $d' = 3/10\pi$  give minima at  $T_R = 3.75$  and  $T_R = 5.00$  respectively.  $ImT$  at the minimum on the other hand depends very weakly on  $d'$  and is an integer. Therefore one can get a large modulus VEV if  $d'$  is small enough, i.e. if there is enough hidden matter which is light enough.

It is well-known that free fermionic strings are formulated at  $T = 1$ , the fixed point of target space duality. The large moduli VEVs mentioned above can only

result if there are untwisted moduli in the string spectrum. In that case, these moduli can obtain VEVs different than unity due to nonperturbative effects in the low-energy supergravity model. This is equivalent to deforming the free fermionic string marginally by adding Abelian Thirring interactions to the string action[21]. The moduli VEVs are related to the couplings of the Abelian Thirring operators which deform the fermionic string marginally. Thus a fermionic string with  $T \neq 1$  arising from hidden gaugino and matter condensation corresponds to a marginally deformed free fermionic string.

For simplicity we considered only one modulus above. Our results can be easily generalized to the more realistic case with more than one modulus of either  $T$  or  $U$  type. In the next section we will see that for acceptable threshold corrections one needs at least three moduli. When there are a number of moduli, matter fields carry modular weights which correspond to each one of them. (With three moduli, the modular weights of matter fields are cyclic permutations of  $(-1/2, -1/2, 0)$  rather than  $-1$ .) As a result, the parameters  $M$  and  $t$  in Eqs. (10b) and (11) are generalized trivially to  $M_i$  and  $t_i$  for each modulus  $T_i$  or  $U_i$ . It is obvious that as long as  $M_i \neq M_j$ ,  $t_i \neq t_j$  for  $i \neq j$ , different moduli will obtain different VEVs from the minimization of  $V_{\text{eff}}$ . In addition, if the corresponding  $d'_i$  are small enough the moduli VEVs will be large as required for large threshold corrections.

#### 4. Gauge coupling unification due to large string threshold corrections

In the previous section we saw how to get large moduli VEVs. In this section we find what is required in order to get threshold corrections of the correct sign and magnitude so that they reconcile the difference between  $M_S$  and  $M_U$ . By the correct sign of threshold corrections we mean  $\Delta_A > 0$  and  $\Delta_B < 0$ .

If there is only one modulus, one cannot obtain the correct sign due to the relation  $\sum_i \frac{1}{2} b_a'^i = b_a$  and the fact that  $\sqrt{ReT}|\eta(T)|^2 < 1$  for all  $T$ . Having more than one modulus does not solve the problem either in realistic models built so far. In these models, the three sectors with  $N = 2$  supersymmetry give one generation

of fermions each so that  $b_1^{ij} = 20/3$ ,  $b_2^{ij} = 0$  and  $b_3^{ij} = -2$  for  $i = 1, 2, 3$ . (In these models, the two Higgs bosons arise from the Neveu–Schwarz sector. Otherwise  $b_2^{ij}$  is different but this does not affect the above conclusion.) It turns out that these values do not give threshold corrections of the correct sign either even if the moduli have large VEVs which are different from each other.

The problem is related to the values of  $b_a^{ij}$  which follow from the equal distribution of MSSM matter in the three  $N = 2$  sectors. Clearly one can obtain different values for  $b_a^{ij}$  for different distributions of matter into the  $N = 2$  sectors. It turns out that a necessary condition for the required corrections is at least one sector with  $b_3^{ij} > 0$  which is not the case in realistic models with the equal division above. Consider now a model in which the MSSM states arise from the three  $N = 2$  sectors in the following (unconventional) manner:  $\{Q_i, u_i, d_i\}$ ,  $\{L_1, e_1\}$ ,  $\{L_2, L_3, e_2, e_3\}$  where each curly bracket denotes a sector. Note that the two Higgs fields are not included. Here we assume that they arise from the Neveu–Schwarz sector of the string as is the case in realistic models. In any case, their presence in any one of the sectors does not change our results qualitatively since they do not affect  $b_3^{ij}$ . This distribution gives

$$b_1^{1j} = 34/3, \quad b_1^{2j} = 3, \quad b_1^{3j} = 17/3, \quad (14a)$$

$$b_2^{1j} = -5, \quad b_2^{2j} = 3, \quad b_2^{3j} = 2, \quad (14b)$$

$$b_3^{1j} = 6, \quad b_3^{2j} = -6, \quad b_3^{3j} = -6 \quad (14c)$$

From the expression for the threshold corrections, Eq. (2), we see that at best there can be three independent contributions to each  $\tilde{\Delta}_a$ ; one from each of the three  $N = 2$  supersymmetric sectors. This requires the presence of at least one modulus (of either  $T$  or  $U$  type) from each of the three sectors. Substituting Eq. (2) and the experimental values of  $\sin^2\theta_w(M_Z)$  and  $\alpha_3(M_Z)$  into Eqs. (3) and (4) we get two equations with four unknowns,  $k, \tilde{\Delta}_1, \tilde{\Delta}_2, \tilde{\Delta}_3$ .  $\tilde{\Delta}_a$  can be traded with the real three unknowns of the problem which are  $T_1, T_2, T_3$ . (The number

of unknowns can increase up to six if  $U$  type moduli are present since they can obtain VEVs different than the  $T$  moduli.)

Here we would like to stress three points. First not for every distribution of MSSM states into sectors there is a solution even though there are two equations and four unknowns. For all distributions with  $b_3^{ii'} < 0$ ,  $i = 1, 2, 3$  and for many with  $b_3^{ii'} > 0$  for some  $i$ , we find that there is no solution since they require  $\sqrt{ReT}|\eta(T)|^2 > 1$  which is not possible. Second, it is very difficult (if not impossible) to find solutions if there are less than three moduli from three different sectors. Third, given a distribution of states into sectors not for every value of  $k_1$  there is a solution. For example, for the above distribution there is no solution if  $k_1 < 1.33$  since this requires  $\sqrt{ReT}|\eta(T)|^2 > 1$ .

From Eq. (3) and (4) we find that for  $k_1 = 1.35$

$$-0.03 = -\tilde{\Delta}_1 + 1.35\tilde{\Delta}_2 \quad (15a)$$

$$-2.37 = \tilde{\Delta}_2 - \tilde{\Delta}_3 \quad (15b)$$

We can translate these into equations for the moduli  $T_{1,2,3}$  by defining (one can include  $U_i$  in this definition if they exist)

$$\log[ReT_{1,2,3}|\eta(T_{1,2,3})|^4] = x, y, z \quad (16)$$

and using the coefficients  $b_a^{ii'}$ . Then Eq. (15) reads

$$-0.03 = -0.54x + 2.78y + 2.77z \quad (17a)$$

$$-2.37 = 0.50x - 1.50y - 2.00z \quad (17b)$$

Every solution of the this set of equations gives a set of moduli VEVs which reconciles the string and MSSM unification scales (for the assumed matter distribution and  $k_1$ ). Thus, for  $k_1 = 1.35$ , we obtain a family of solutions which is fixed by any

of  $x, y, z$ , say  $z$ . (Note that  $x, y, z \leq -1.06$  by definition. The maximum is obtained at the fixed point  $T_i = 1$ .) For example, taking  $z = -2.15$  gives  $y = -1.10$  and  $x = -16.64$  whereas  $z = -1.20$  gives  $y = -1.60$  and  $x = -14.34$ . We find that large moduli VEVs are needed to obtain these values. For example, the first set of solutions corresponds to  $T_1 \sim 18.6, T_2 \sim 1.3, T_3 \sim 3.1$  whereas the second set is given by  $T_1 \sim 16.4, T_2 \sim 2.3, T_3 \sim 1.6$ . (If there are also  $U$  moduli present in some sectors, the corresponding VEVs for  $T_i$  are smaller than the above values depending on the VEVs of  $U$  moduli.) We saw in the previous section that these large moduli VEVs can be naturally obtained in a supersymmetry breaking scenario due to both hidden gaugino and matter condensation.

Above we found that there is no solution for  $k_1 < 1.33$  whereas very close to this value (i.e.  $k_1 = 1.35$ ) there is a realistic solution. As  $k_1$  increases, solutions continue to exist but they require very large values of  $T_i$ . For example for  $k_1 = 1.5$  we need at least one  $T_i \sim 100$  which is impossible to obtain in the supersymmetry breaking scenario considered above since this requires  $d' \sim 1/200$  which cannot be obtained for realistic values of the parameters  $N, M, t$ . Thus for the MSSM matter distribution we considered above realistic solutions which resolve the difference between  $M_S$  and  $M_U$  require loosely  $1.33 < k_1 < 1.45$ . The lower bound arises from the conditions  $x, y, z < 0$  whereas the loose upper bound is due to the fact that  $d'$  cannot be smaller than a minimal value around  $1/120$ .

We considered a specific distribution of MSSM states into the three  $N = 2$  sectors for concreteness. There are other similar distributions which give solutions with the same properties as long as there is one sector with  $b_3^{ii} > 0$ . For example, any different distribution of leptons into the sectors gives another solution. Generically, we need a matter distribution with  $b_3^{ii} > 0$  for some  $i$ . This will give a one parameter family ( $z$  in our case) of solutions for all  $k_1$  between some minimal and maximal values (1.33 and  $\sim 1.45$  in our case).

## 5. Conclusions

In this letter, we discuss a way to reconcile the MSSM and string unification scales or to obtain acceptable  $\sin^2\theta_w(M_Z)$  and  $\alpha_3(M_Z)$  from the string (with only the MSSM spectrum) by considering string threshold corrections. This requires the presence of at least three untwisted moduli from the three  $N = 2$  supersymmetric sectors of the string spectrum. The moduli VEVs must be large and different from each other in order to get large corrections. Large moduli VEVs can be naturally obtained in a scenario with gaugino and matter condensation leading to supersymmetry breaking. This corresponds to a fermionic string which is marginally deformed by Abelian Thirring interactions whose couplings are related to the moduli VEVs. In addition, MSSM states must be distributed into the  $N = 2$  sectors in an unconventional manner to obtain threshold corrections of the correct sign. Such a distribution must satisfy  $b_3'^i > 0$  for some sector  $i$ . We gave a representative example of such a distribution with the required moduli VEVs. We found a one parameter (any of the moduli) family of solutions for every value of  $k_1$  between some maximal and minimal values depending on the matter distribution. There are other distributions with at least one  $b_3'^i > 0$  which give solutions with similar properties. Whether any of required distributions of states can arise in realistic free fermionic string models needs to be seen in explicit model building attempts. We assumed throughout the paper that the contributions to the threshold corrections which do not depend on the moduli are negligible, i.e.  $c_a, Y \ll \tilde{\Delta}_a$  which holds for known models. If this is not the case the required moduli VEVs will be larger or smaller depending on the sign of these terms.

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## REFERENCES

1. U. Amaldi et al, Phys. Rev. D **36** (1987) 1385; P. Langacker and M. Luo, Phys. Rev. D **44** (1991) 817; J. Ellis, S. Kelley and D. V. Nanopoulos, Phys. Lett. B **249** (1990) 441.
2. M. B. Green, J. H. Schwarz and E. Witten, *Superstring Theory, Vols. 1 and 2* (Cambridge University Press, Cambridge 1987).
3. V. Kaplunovsky, Nucl. Phys. B **307** (1988) 145.
4. K. R. Dienes and A. E. Faraggi, preprint IASSNS-HEP-95/24, hep-th/9505018.
5. T. Kobayashi, D. Suematsu and Y. Yamagishi, Phys. Lett. B **329** (1994) 27.
6. L. J. Dixon, V. Kaplunovsky and J. Louis, Nucl. Phys. B **355** (1991) 649.
7. J. P. Deredinger, L. E. Ibanez and H. P. Nilles, Phys. Lett. B **155** (1985) 65; M. Dine, R. Rohm, N. Seiberg and E. Witten, Phys. Lett. B **156** (1985) 55.
8. I. Antoniadis, C. Bachas, and C. Kounnas, Nucl. Phys. B **289** (1987) 87; I. Antoniadis and C. Bachas, Nucl. Phys. B **298** (1988) 586; H. Kawai, D.C. Lewellen, and S.H.-H. Tye, Nucl. Phys. B **288** (1987) 1.
9. L. E. Ibanez, Phys. Lett. B **318** (1993) 73.
10. E. Halyo, Nucl. Phys. B **438** (1995) 138.
11. I. Antoniadis, J. Ellis, R. Lazace and D. V. Nanopoulos, Phys. Lett. B **268** (1991) 188.
12. Particle Data Group, Phys. Rev. D **50** (1994) .
13. A. Font, L. Ibanez, D. Lust and F. Quevedo, Phys. Lett. B **245** (1990) 401.
14. I. Antoniadis, Phys. Lett. B **246** (1990) 377.
15. E. Halyo, Mod. Phys. Lett. **A9** (1994) 1415.
16. S. Ferrara, N. Magnoli, T. Taylor and G. Veneziano, Phys. Lett. B **245** (1990) 409; D. Lüst and T. Taylor, Phys. Lett. B **253** (1991) 335.

17. D. Amati *et. al.*, Phys. Rep. **162** (1988) 169.
18. A. E. Faraggi and E. Halyo, preprint IASSNS-HEP-94/17, hep-ph/9405223.
19. E. Halyo, preprint WIS-95/17/MAR-PH, hep-th/9505214.
20. E. Halyo, Phys. Lett. B **343** (1995) 161.
21. D. Chang and A. Kumar, Phys. Lett. B **211** (1988) 76; Phys. Rev. D **38** (1988) 1893.